Math 241, Spring 2022 — Period doubling bifurcation

Class on February 24

In this set of problems we get introduced to the *period doubling bifurcation* of the family of maps $Q_c(x) = x^2 + c$. When this bifurcation takes place, our dynamical system transitions from having just fixed points to having both fixed points and period-2 points.

Problem 1. Consider the equation $Q_c^2(x) = x$.

1. Use the quadratic formula to find solutions to the equation.
2. For which range of values of c does this equation have two solutions? Three solutions? Four solutions? The c value when this transition happens is called the period doubling bifurcation value.
3. Give formulas for the period-2 points (when the exist).

Problem 2. Let q_- and q_+ be the left and right period-2 points. For which range of c values do these points form an attracting 2-cycle? Neutral? Repelling?

Problem 3. Write a summary of your findings.

1. State the range of c values when there are no period-2 points.
2. State the range of c values when there are period-2 points that are attracting.
3. State the range of c values when there are period-2 points that are repelling.

Problem 4. The saddle node bifurcation at $c = 1/4$ was straightforward to visualize using graphs of $y = Q_c(x)$ and $y = x$, but the period doubling bifurcation isn't so apparent at first. Try graphing $y = Q_c^2(x)$ and $y = x$ for various values of c around $c = -3/4$. What do you notice? What do you notice around $c = -5/4$?