Math 241, Spring 2022 — Chaos

Class on April 12

We have learned that Devaney defines a dynamical system $F : X \to X$ to be *chaotic* if

1. periodic points of F are dense in X
2. F is transitive
3. F has a sensitive dependence on initial conditions.

Problem 1. For each of the following dynamical systems, discuss whether you believe the system has any of the three properties above. Give intuition for why you believe it's true or explain what would need to be proven to back up your intuition.

1. $F(x) = 2x(1-x), X = [0,1]$
2. $T(x) = \begin{cases} 3x & 0 \le x \le 1/2 \\ 3-3x & 1/2 < x \le 1 \end{cases}, X = K$, the Cantor middle-thirds set