

Math 241, Spring 2022 — Neutral fixed points

Class on February 15

In this set of problems we get practice understanding what happens to orbits that start near neutral fixed points.

Problem 1. Each of the the following functions has a neutral fixed point p . As a result, the behavior of orbits that start nearby might converge toward the fixed point (we call p a *weakly attracting* fixed point), repel away from it (we call p a *weakly repelling* fixed point), or have both behaviors depending on which side of p the orbit starts (we don't have a name for this). Determine the behavior of orbits that start near the neutral fixed point using a cobweb diagram.

1. $F(x) = x^3 + x$
2. $F(x) = x^2 - x$

Problem 2. Suppose F has a neutral fixed point at p with $F'(p) = 1$. Suppose also that $F''(p) > 0$.

1. Make a sketch of the graph of F near p for a hypothetical function with the properties above.
2. Based on your sketch, what can you say about p : is it weakly attracting, weakly repelling, or neither?
3. What happens when $F''(p) < 0$?