Math 301, Spring 2023 — Exam 2

Mount Holyoke College

Due May 8 at 12:00 pm on Gradescope

Instructions. The take-home portion of the exam consists of 2 questions for a total of 20 points. The first problem is open ended but full credit will be given to well written, thoughtful responses that are half of a page to a page long. To receive full credit on the second problem, you must write clearly and provide sufficient justifications. LaTeX is not required. Before proceeding to the problems, please read all of the instructions.

Please feel free to ask me questions (over email or private Piazza post) if you have doubts about anything. You may use your book, notes, and any class materials that appear on the class web page, but you may not consult with each other on any aspect of the class or with outside resources like the TA, friends, family, other textbooks, or internet posts. I know that you're all working hard, both in this class and outside of it. I want you to be proud of the effort that you've put in, proud of your individual growth, and proud of your integrity. Part of that means taking the honor code seriously, and working on this exam by yourself. I write all of this not because I suspect that you'll cheat, but because I want you to know that I value you each as individuals and value your work and ideas, right or wrong. *At the top of your submission, write me a short note acknowledging that you've read this and understand the restrictions on the exam.*

Unless stated otherwise, you may use any definitions or theorems that we've discussed in lecture or homework. These are not meant to be trick questions with one-liner proofs, but rather straightforward questions that ask you to get your hands dirty with definitions and details.

Problem 1. What do you see as the story we've told in the course? Are there common themes or through lines? Can you connect the first half of the course, which had a focus on sequences, with the second half which had a focus on functions? What ideas in the course have you found most interesting in the course? Why? Feel free to answer only a (non-empty) subset of these questions.

Problem 2. A set $\mathcal{F}$ of functions with domain $[0,1]$ is said to have *Property E* if for every $\epsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - f(y)| < \epsilon$ when $x, y \in [0,1]$ and $|x - y| < \delta$ for all $f \in \mathcal{F}$.

1. Let $N \geq 1$ and let $\mathcal{F} = \{f_1, \ldots, f_N\}$ be a finite collection of continuous functions on $[0,1]$. Prove that $\mathcal{F}$ has Property E.

2. For each $n \geq 1$, define $f_n(x) = x^n$. Let $\mathcal{F} = \{f_n : n \geq 1\}$ be the infinite collection of such functions. Prove that $\mathcal{F}$ has does not have Property E.