

Math 301, Spring 2025 — Homework 2

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Due February 14 at 5:00 pm

Instructions. This problem set contains problems mostly from Week 2 of class. The problem numbers refer to our textbook, *Understanding Analysis* by Stephen Abbott.

Problem 1. Please do the following textbook problems: Exercises 1.3.1, 1.3.2, 1.3.5a, 1.4.2, 1.4.4, 1.5.5, 1.5.4a

Remark. For Exercise 1.5.4a, you can use Exercise 1.5.5. You can also use the fact that $f : (-\pi/2, \pi/2) \rightarrow \mathbb{R}$ given by $f(x) = \tan x$ is a bijection. Finally, try making a bijection $g : (-\pi/2, \pi/2) \rightarrow (a, b)$. *Hint: make a line connecting the point $(-\pi/2, a)$ to the point $(\pi/2, b)$.*

Problem 2. Let $S, T \subseteq \mathbb{R}$ be nonempty, bounded sets with the property $S \subseteq T$. Prove that $\inf T \leq \inf S$.

Problem 3. Let $S, T \subseteq \mathbb{R}$ be nonempty sets with the property that $s \leq t$ for all $s \in S$ and $t \in T$.

- a. Prove that S is bounded above and T is bounded below.
- b. Prove that $\sup S \leq \inf T$.
- c. Give an example where $\sup S = \inf T$ and $S \cap T = \emptyset$.