Math 301, Fall 2021 — Homework 6

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Due October 22

Instructions. Please submit your solutions to the following problems on Gradescope. For the group part, please type your answers in LaTeX and submit the output PDF. You'll only submit one write-up for the whole group. For the solo part, you may handwrite solutions or use LaTeX. Make sure to select which problem is on each page in Gradescope.

Group problem

No group problem this week.

Solo problems

Problem 1. For each of the following series, determine whether it converges and justify your answer.

1. $\sum_{n=1}^{\infty} \frac{n+3}{4n+7}$
2. $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n \left(1+\frac{5}{n}\right)$ *Hint: use the comparison test.*
3. $\sum_{n=1}^{\infty} \frac{n-1}{n^2}$ *Hint: do a little algebra, use the fact that $n^2 \geq 2n$ for all $n \geq 2$, and use the comparison test.*

Problem 2. Use the Cauchy criterion to prove that if $\sum |a_n|$ converges and (b_n) is bounded, then $\sum a_n b_n$ converges.

Problem 3. Prove that if $a_n \geq 0$ and $\sum a_n$ converges, then $\sum a_n^2$ converges. *Hint*: use the previous problem.

Problem 4. Let (a_n) be a sequence such that

$$|a_{n+1} - a_n| < 2^{-n}$$

for all $n \geq 1$. Prove that (a_n) is a Cauchy sequence. *Hint*: remember that $\sum 2^{-n}$ converges since it's a geometric series, so it satisfies the Cauchy criterion for series.