

Math 301, Spring 2023 — Homework 6

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Due March 24

Instructions. Please submit your solutions to the following problems on Gradescope. Your proof answers should be written in complete sentences and avoid using symbols like \Rightarrow , \therefore , or \because . Edit rough drafts and reread the guidelines for writing mathematics before submitting. **No LaTeX required this week.** When you submit handwritten solutions, make sure your scan is clear, well-aligned, and as readable as possible. Make sure to select which problem is on each page in Gradescope.

Problem 1. The alternating series test is very useful for proving alternating series converge, but there are often other convergence tests that can be used instead. All of the series below converge, but only one *requires* the alternating series test to prove convergence. For each series below, prove it converges but you may only use the alternating series test for one of the series.

- a. $\sum (-1)^{n+1} \frac{2^n}{n!}$
- b. $\sum (-1)^{n+1} \frac{1}{n^{1/2}}$
- c. $\sum (-1)^{n+1} \frac{1}{n^3}$
- d. $\sum (-1)^{n+1} \frac{1}{3^n}$

Problem 2. For each description below, give an example of a series that fits the description or explain why such an example is not possible.

- a. A divergent series $\sum a_n$ for which $\sum a_n^2$ converges.
- b. A convergent series $\sum a_n$ for which $\sum a_n^2$ diverges.
- c. A convergent series $\sum a_n$ with positive terms for which $\sum a_n^2$ diverges.

Problem 3. Determine the values of $p > 0$ for which the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converges.

Problem 4. Let $A = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$.

- a. Give a bound on the error in using the 50th partial sum of $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ to approximate A .
- b. Find the smallest value of n so that the n th partial sum of $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ is within 0.001 of A .