

Math 301, Spring 2023 — Homework 8

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Due April 7

Instructions. Please submit your solutions to the following problems on Gradescope. Your proof answers should be written in complete sentences and avoid using symbols like \Rightarrow , \therefore , or \because . Edit rough drafts and reread the guidelines for writing mathematics before submitting. **Please typeset Problem 1 with LaTeX.** When you submit handwritten solutions, make sure your scan is clear, well-aligned, and as readable as possible. Make sure to select which problem is on each page in Gradescope.

Problem 1. In class we proved that if $f : [a, b] \rightarrow \mathbb{R}$ is continuous, then it achieves its maximum at some point $y_0 \in [a, b]$. Mimic that proof to show that f also achieves its minimum at some point $x_0 \in [a, b]$.

Problem 2. Let $f(x) = x^5 - 5x^4 + 11x^3 - 12x^2 + 7x - 1$. Prove that f has at least one real root.

Problem 3. Prove that the equation $x = \cos x$ has at least one solution in the interval $(0, \pi/2)$. You may assume that $f(x) = \cos x$ is a continuous function.

Problem 4. Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function and $f(a)f(b) < 0$ for some $a, b \in \mathbb{R}$. Prove that there exists x between a and b such that $f(x) = 0$.

Problem 5. For each of the following functions $f : D \rightarrow \mathbb{R}$ prove that f is uniformly continuous on D by giving an ϵ - δ proof.

- a. $f(x) = 3x + 11, D = \mathbb{R}$
- b. $f(x) = x^3, D = [-2, 1]$
- c. $f(x) = 1/x, D = [1/2, \infty)$
- d. $f(x) = \frac{5x}{2x-1}, D = [1, \infty)$