Announcements

- Quiz 3 posted after class
  - due at 3pm tomorrow
- HW 3 due Friday

Problem 2. The monotone convergence theorem can be used to prove series converge, which is useful since we often don't know the value of infinite sums. Here, we outline a proof that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges. Let $m \geq 1$ and consider the *partial sum sequence*

$$s_m = \sum_{n=1}^{m} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \cdots + \frac{1}{m^2}.$$

Recall that a series converges if its partial sum sequence converges. That is, if $\lim s_m$ exists, then $\sum \frac{1}{n^2}$ converges.

a. Explain why (s_m) is increasing.

b. It's straightforward to see that $n^2 > n(n-1)$ and, using a little algebra, it's possible to show that

$$\frac{1}{n(n-1)} = \frac{1}{n-1} - \frac{1}{n}$$

for all $n \geq 1$. Use these facts to show that (s_m) is bounded above by 2. If you get stuck and need a reminder about the idea of a telescoping series, ask me.

c. Make a conclusion.

(a) $s_{m+1} \geq s_m$ is clear since we're summing positive values

(b) $$s_m = \sum_{n=1}^{m} \frac{1}{n^2}$$

$$= 1 + \sum_{n=2}^{m} \frac{1}{n^2}$$

$$< 1 + \sum_{n=2}^{m} \frac{1}{n(n-1)}$$

$$= 1 + \sum_{n=2}^{m} \left(\frac{1}{n-1} - \frac{1}{n}\right)$$

$$= 1 + \left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \dots + \left(\cancel{\frac{1}{m-2}} - \cancel{\frac{1}{m-1}}\right) + \left(\cancel{\frac{1}{m-1}} - \frac{1}{m}\right)$$

$$= 2 - \frac{1}{m} < 2$$

holds for all m.

By MCT our sequence (s_m) converges, by the order limit theorem the limit ≤ 2.

After class

Read theorems 11.3, 11.4, and 11.5.