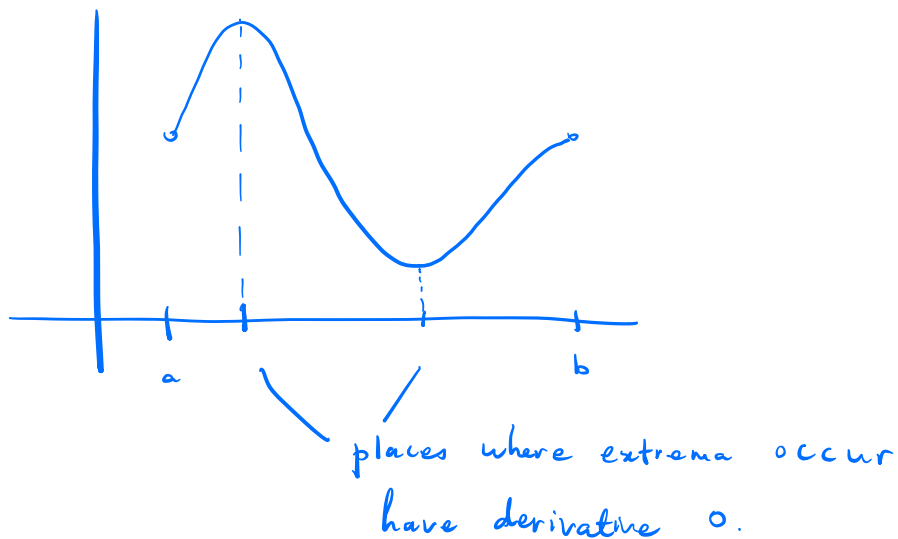


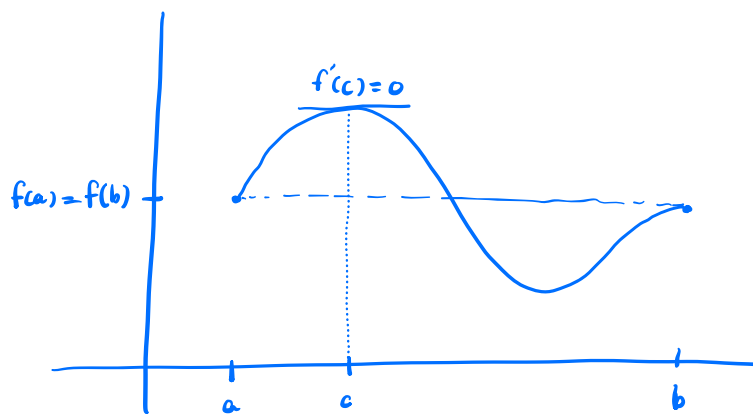
## § 29 The mean value theorem

Today we'll study three fundamental results in the theory of differentiable functions. The culminating result, the Mean Value Theorem, is very important since it's used to understand theorems like the Fundamental Theorem of Calculus and Taylor's theorem.

Theorem (Interior extremum theorem) Let  $f: (a, b) \rightarrow \mathbb{R}$  be a given function and let  $c \in (a, b)$ . If  $f$  achieves a max/min at  $c$  and  $f$  is differentiable at  $c$ , then  $f'(c) = 0$ .



Theorem (Rolle's theorem) Let  $f$  be continuous on  $[a, b]$  and differentiable on  $(a, b)$ . If  $f(b) = f(a)$  then  $\exists c \in (a, b)$  such that  $f'(c) = 0$ .



There must be a place where slope is 0 since secant slope is 0.

Theorem (Mean value theorem). Let  $f$  be continuous on  $[a, b]$  and differentiable on  $(a, b)$ . Then  $\exists c \in (a, b)$  such that  $f'(c) = \frac{f(b) - f(a)}{b - a}$ .

