

Math 301 — Quiz 4

Summary. Please do the following problems. You may use your book, notes, and other class materials, but you may not consult with each other or use any other resources. You may use LaTeX or handwrite your solutions. Please do whichever you prefer.

Problem 1. Let $x_1 = 2$ and for all $n \geq 1$, define

$$x_{n+1} = \frac{1}{2} \left(x_n + \frac{2}{x_n} \right).$$

- Find the first 4 terms of the series.
- Assume the sequence converges. Find its limit by solving an algebraic equation.
- Prove the sequence converges using the monotone convergence theorem.

Problem 2. Recall the following order limit theorem, a version of which was proved in class/homework: if (s_n) converges to L and $s_n \geq a$ for all $n \geq 1$, then $L \geq a$. Explain whether the following modification is a true or false statement: if (s_n) converges to L and $s_n > a$ for all $n \geq 1$, then $L > a$. If it's true, prove it; if it's false, give a counterexample.