Math 301 — Induction

Problem 1. Consider the inequality $n^2 > n + 1$. Make a conjecture for the values of $n \in \mathbb{N}$ for which the inequality holds and use induction to prove your conjecture.

Problem 2. Consider the inequality $n! > n^2$. Make a conjecture for the values of $n \in \mathbb{N}$ for which the inequality holds and use induction and the previous problem to prove your conjecture.

Problem 3. Let $x_1 = 2$ and define $x_{n+1} = 3^{-1}(x_n + 5)$ for all $n \in \mathbb{N}$.

a. Use induction to prove that $x_n \leq x_{n+1}$ for all $n \in \mathbb{N}$.

b. Use induction to prove that $x_n < 3$ for all $n \in \mathbb{N}$.