

Math 301 — Subsequences

Problem 1. Consider the Fibonacci sequence $(a_n) = (1, 1, 2, 3, 5, 8, 13, 21, 34, \dots)$. Determine whether each of the following are subsequences of (a_n) .

- a. $(1, 3, 8, 21, 55, \dots)$
- b. $(1, 2, 1, 5, 3, 13, 8, 34, 21, \dots)$
- c. $(1, 1, 2, 3, 5, 8, 13, 21, 34, \dots)$
- d. $(1, 1, 1, 1, 2, 2, 3, 3, 5, 5, 8, 8, 13, 13, 21, 21, 34, 34, \dots)$

Problem 2. Make a conjecture about whether each of the following statements is true or false. If you believe a statement is false try to give a counterexample.

- a. If some subsequence of (a_n) converges to L then (a_n) converges to L .
- b. If every subsequence of (a_n) converges to L then (a_n) converges to L .
- c. If (a_n) converges to L then every subsequence of (a_n) converges to L .
- d. Every sequence contains a convergent subsequence.
- e. Every monotone sequence contains a convergent subsequence.
- f. Every bounded sequence contains a convergent subsequence.