Math 301 — Cauchy sequences

Summary. A sequence (a_n) is called *Cauchy* or a *Cauchy sequence* if for any $\epsilon > 0$ there exists $N > 0$ such that $|a_n - a_m| < \epsilon$ whenever $n, m > N$.

Problem 1. Discuss the following questions. Assume (a_n) is a sequence of real numbers.

a. T/F: If (a_n) is bounded then it is Cauchy.

b. T/F: If (a_n) is monotone then it is Cauchy.

c. T/F: If (a_n) is convergent then it is Cauchy.

d. T/F: If (a_n) is Cauchy then it is convergent.

e. T/F: If (a_n) is Cauchy, then it has a convergent subsequence.

f. T/F: $a_n = 1/n$ is a Cauchy sequence.

g. T/F: If (a_n) is Cauchy and $a_n \in \mathbb{Q}$ for all $n \geq 1$ then it converges to a rational number.

h. T/F: If (a_n) is Cauchy and $a_n \in (0, 1)$ for all $n \geq 1$ then it converges to an element of $(0, 1)$.