Math 301 — Series

Problem 1. For each of the following series, decide whether it converges, giving a justification based on one of the tests we've introduced along with the convergence of our baseline examples (geometric series and p-series). These are the warm-up problems

a. $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$

b. $\sum_{n=1}^{\infty} \frac{1}{2^n+n}$

c. $\sum_{n=2}^{\infty} \frac{1}{\ln n}$

d. $\sum_{n=1}^{\infty} \frac{n}{n+1}$

e. $\sum_{n=2}^{\infty} \frac{n^2}{n^3-1}$

Problem 2. Take as given that p-series converge when $p > 1$ and alternating p-series converge for all $p > 0$. Make a conjecture about whether each of the following statements is true or false and give a brief explanation or counterexample as justification.

a. If (a_n) is a Cauchy sequence, then the series $\sum a_n$ converges.

b. If the series $\sum a_n$ converges, then (a_n) is a Cauchy sequence.

c. If the series $\sum a_n$ converges, then the series $\sum |a_n|$ converges.

d. If the series $\sum |a_n|$ converges, then the series $\sum a_n$ converges.