

## Math 301 — Ratio test

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**Problem 1.** Determine whether the following statement is true or false; justify or give a counterexample. *If  $\sum |a_n|$  diverges, then  $\sum a_n$  diverges.*

**Problem 2.** Consider the series  $\sum a_n$ , let  $L = \lim_{n \rightarrow \infty} |a_{n+1}/a_n|$ . Let's try to prove the second statement of the ratio test: *if  $L > 1$ , then  $\sum a_n$  diverges.*

- Adapt the steps of the proof for the case  $L < 1$  to the case  $L > 1$ .
- What goes wrong with your adaptation of the proof if you try using the comparison test to compare with a divergent geometric series? Where does the logic fail?
- Fix your proof using the Test for Divergence.

**Problem 3.** Let's try to prove the third statement of the ratio test: *if  $L = 1$ , then the ratio test is inconclusive.*

- Find a convergent series where  $L = 1$ .
- Find a divergent series where  $L = 1$ .
- Explain why your two examples prove the third statement of the ratio test.

**Problem 4.** Explain whether the following series converge.

- $\sum_{n=1}^{\infty} \frac{n^5}{7^n}$
- $\sum_{n=1}^{\infty} \frac{n^3}{n!}$
- $\sum_{n=1}^{\infty} \frac{n!}{2^n}$