Math 301 — Integral test and p-series

Problem 1. Let's try to prove $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

a. Sketch a graph of the function $f(x) = 1/x^2$.

b. Relate the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ to the area under the graph of $f(x)$ over the interval $[1, \infty)$. *Be careful here; it's slightly more subtle than our first example.*

c. Compute $\int_1^{\infty} f(x)\,dx$.

d. Make a conclusion using the Monotone Convergence Theorem.

Problem 2. The *Integral Test* states that if f is positive and decreasing on the interval $[1, \infty)$, $\lim_{x\to\infty} f(x) = 0$, and $a_n = f(n)$ for all $n \geq 1$, then $\sum_{n=1}^{\infty} a_n$ converges if and only if $\int_1^{\infty} f(x)\,dx$ is finite.

a. Compute $\int_1^{\infty} \frac{1}{x^p}\,dx$ for any constant $p > 0$. Your answer should be a formula in terms of p.

b. Use the Integral Test to find the values of $p > 0$ for which the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges.