Math 301 — Alternating series

Problem 1. Suppose (a_n) is a decreasing sequence of positive numbers that converges to 0. Let (s_n) be the partial sum sequence of $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$. Prove the following claims.

a. The subsequence (s_{2k-1}) is decreasing.

b. The subsequence (s_{2k}) is increasing.

c. The two subsequences above converge.

d. $\lim_{k\to\infty}(s_{2k-1} - s_{2k}) = 0$.

e. The two subsequences above converge to the same value A.

f. The sequence (s_n) converges to A.

g. $|s_n - A| \leq a_n$

Problem 2. Consider the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}}$.

a. Explain why the series converges.

b. Let A be the value the series converges to. Suppose we try to approximate L using the sum of the first 10 terms of the series. Give a bound on the error of this approximation.

c. Suppose we want to approximate A by summing the first n terms of the series. How big should n be so that the error of this approximation is no more than 0.0001?