Math 301 — Basic inequalities

Problem 1. Let $a, b, c, d \in \mathbb{R}$. Give brief proofs for the following inequalities.

a. $|a - b| \leq |a| + |b|$

b. $|a + b + c| \leq |a| + |b| + |c|$

c. $|a - b| \leq |a - c| + |c - d| + |d - b|$

Problem 2. Decide which of the following statements is true. Give a brief justification if the statement is valid and a counterexample if it is not.

a. Two real numbers a, b satisfy $a < b$ if and only if $a < b + \epsilon$ for every $\epsilon > 0$.

b. Two real numbers a, b satisfy $a \leq b$ if and only if $a < b + \epsilon$ for every $\epsilon > 0$.

Problem 3. Write the negation of the following statement and then make a guess about whether the statement itself or its negation is true.

There exists a real number $x > 0$ such that $x < 1/n$ for all $n \in \mathbb{N}$.