Math 301 — Continuity

Problem 1. Give an ϵ-δ proof showing that $f(x) = x^3$ is continuous.

Problem 2. Give an ϵ-δ proof showing that $f(x) = \begin{cases} x & x \in \mathbb{Q} \\ 0 & x \notin \mathbb{Q} \end{cases}$ is continuous at 0.

Problem 3. Pove that $f(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & x \notin \mathbb{Q} \end{cases}$ is not continuous at 0. You may assume the fact that $\sqrt{2}/n$ is irrational for all $n \geq 1$.

Problem 4. Let f and g be functions that are continuous at a. Prove the following functions are continuous at a as well:

a. $f + g$

b. fg