Math 301 — Extreme value theorem

Problem 1. Make a conjecture about whether each of the following statements is true or false. If you believe a statement is false, give a counterexample.

a. If $f : (0, 1) \to \mathbb{R}$ is continuous, it attains a maximum and minimum value.

b. If $f : (0, 1) \to \mathbb{R}$ is continuous and bounded, it attains a maximum and minimum value.

c. If $f : [0, 1] \to \mathbb{R}$ is continuous, it attains a maximum value.

Problem 2. Consider the function $f(x) = x^3 - 6x^2 + 9x + 1$. Does this function have a maximum and minimum on $[0, 5)$? What about $[0, 5]$? Use calculus to determine the maximum and minimum values if they exist.

Problem 3. How does the proof of the Extreme Value Theorem break down if $[a, b]$ is replaced with (a, b)?