Math 301 —Intermediate value theorem

Problem 1. Let $f(x) = x^5 - 2x^3 - 2$. Prove that the equation $f(x) = 0$ has at least one solution.

Problem 2. Prove that the equation $xe^x = 2$ has at least one solution. You may assume that $f(x) = e^x$ is a continuous function.

Problem 3. Let $f, g : [a, b] \to \mathbb{R}$ be continuous functions on the interval $[a, b]$ such that $f(a) \geq g(a)$ and $f(b) \leq g(b)$. Prove that $f(x_0) = g(x_0)$ for at least one point $x_0 \in [a, b]$.