

## Math 301 — Consequences of the Mean Value Theorem

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**Problem 1.** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be given and suppose that  $|f(x) - f(y)| \leq (x - y)^2$  for all  $x, y \in \mathbb{R}$ . Prove that  $f$  is a constant function.

**Problem 2.** Let  $f : I \rightarrow \mathbb{R}$  be given. Suppose that  $f$  is twice differentiable on  $I$  and  $f''(x) = 0$  for all  $x \in I$ . Prove that  $f(x) = ax + b$  for some  $a, b \in \mathbb{R}$ .

**Problem 3.** Prove that  $\sin x \leq x$  for all  $x \geq 0$ .