

## Math 301 — Suprema and infima

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**Problem 1.** For each set below, give its maximum and minimum (if they exist), as well as its supremum and infimum.

- a. The half open interval  $(-2, 4] = \{x \in \mathbb{R} : -2 < x \leq 4\}$ .
- b.  $\{1/n : n \in \mathbb{N}\}$
- c.  $\bigcap_{n=1}^{\infty} (-1/n, 1 + 1/n)$
- d.  $\{r \in \mathbb{Q} : 0 \leq r^2 \leq 2\}$
- e.  $\{r \in \mathbb{R} : 0 \leq r^2 \leq 2\}$

**Problem 2.** Let  $S \subseteq \mathbb{R}$  be a non-empty set that is bounded above and let  $\alpha \in \mathbb{R}$  be an upper bound for  $S$ . Prove the statement below. Note that we proved the converse in lecture.

If for every  $\epsilon > 0$  there exists  $x \in S$  such that  $x > \alpha - \epsilon$ , then  $\alpha = \sup S$ .

**Problem 3.** Let  $S$  and  $T$  be non-empty, bounded subsets of  $\mathbb{R}$  and suppose that  $S \subseteq T$ . Prove that  $\sup S \leq \sup T$ .