

Math 301 — Computational approach to uniform convergence

Problem 1. Let $f_n : [0, \infty)$ be given by

$$f_n(x) = \frac{nx}{1 + n^2x^2}$$

for all $n \geq 1$.

- Find $f(x) = \lim_{n \rightarrow \infty} f_n(x)$ for all $x \in [0, \infty)$.
- Determine whether $f_n \rightarrow f$ uniformly on $[0, 1]$.
- Determine whether $f_n \rightarrow f$ uniformly on $[1, \infty)$.

Problem 2. Repeat the previous problem with

$$f_n(x) = \frac{x}{1 + nx^2}.$$