

Math 301 — More on sequences

Summary. We should think of proving that a sequence (a_n) converges to a value L as a challenge that must be satisfied: a challenger has given us a tiny number $\epsilon > 0$ and we must come up with a value N so that a_n is within ϵ units of L when $n > N$. That is, we need to find how far along in the sequence we must go so that we're within an ϵ -“neighborhood” of L .

Problem 1. The examples from our first day of sequences were hopefully straightforward, at least up to a little bit of algebra to be done. The examples in this problem ask you do similar proofs, but finding N takes a little more work. These are more like Example 3 in Section 8.

a. $a_n = \frac{n^2+3}{n^2-3}, L = 1$

b. $a_n = \frac{n^2-3n+2}{n^2+3}, L = 1$

Problem 2. Showing a sequence diverges often times requires a proof by contradiction. Try emulating the proof that $a_n = (-1)^n$ diverges (Example 4 in Section 8) to show that $a_n = \sin(n\pi/2)$ diverges.