Math 301 — Limit theorems

Problem 1. Let (a_n) be a convergent sequence and let $L = \lim_{n\to\infty} a_n$. Suppose $k \neq 0$. Prove that $\lim ka_n = kL$. (*Remark: this fact is of course true when* $k = 0$ *also but you need not consider that case here.*)

Problem 2. Let (a_n) and (b_n) be convergent sequences and let $L_1 = \lim_{n\to\infty} a_n$ and $L_2 = \lim_{n\to\infty} b_n$. Prove that $\lim_{n\to\infty}(a_n + b_n) = L_1 + L_2$.

Problem 3. Let (a_n) be a convergent sequence and let $L = \lim_{n\to\infty} a_n$. Suppose $a_n > c$ for all but finitely many $n \geq 1$. Prove that $L \geq c$.

Problem 4. Give an example of a convergent sequence (a_n) where $a_n > 0$ for all but finitely many $n \geq 1$ whose limit L fails to satisfy the strict inequality $L > 0$. This example is meant to explain why we *need* the non-strict inequality in the theorem you've proved in Problem 3.