Math 339SP, Spring 2024 — Homework 2

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Due February 9 at 5:00 pm

Instructions. This problem set covers material from Week 2 of class, with a focus on Chapter 2 of the textbook.

Problem 1. Try the following exercises from Chapter 2.

1. Exercise 2.4
2. Exercise 2.6b
3. Exercise 2.16 (I want you to do a special case of this problem and just show that $P^2 = P$. The general case of $P^n = P$ can be proved by induction and you can try this for fun but it's not required. Note that a stochastic matrix is one with the property that, for any given row, its entries sum to 1.)
4. Exercise 2.18 (Note that you're assuming that the initial distribution is $\alpha = (1/k, 1/k, \ldots, 1/k)$ and you're trying to prove that $P(X_n = j) = 1/k$ for any $j = 1, \ldots, k$.)
5. Exercise 2.26

Problem 2. Consider a 3-state Markov chain which is constructed as follows. Let

$$Q = \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix}, \quad R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

be 3×3 stochastic matrices (which correspond to two other Markov chains with the same state space). Before each step of our Markov chain, we toss a fair coin. If it lands heads, we will transition to a new state according to the probabilities in Q; if it lands tails, we will use R.

1. Give the transition matrix P of our Markov chain, entry by entry, in terms of the entries of Q and R.
2. Express P as a linear combination of Q and R. That is, find constants α and β so that $P = \alpha Q + \beta R$.
3. Modify your previous answer for the following case: we use a biased coin with heads probability $p \neq 1/2$ and when the coin lands tails, instead of transitioning according to the probabilities in R, we stay in our current state.

Problem 3. Do Problem 3 from the worksheet on Thursday, February 1.