

Math 339SP, Spring 2024 — Homework 4

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Due February 23 at 5:00 pm

Instructions. This problem set covers material from Week 4 of class, with a focus on Chapter 3 of the textbook.

Problem 1. Consider the Markov chain with state space $\mathcal{S} = \{1, 2, 3, 4, 5, 6\}$ and transition matrix

$$P = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0.3 & 0.7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0.9 & 0 \\ 0.25 & 0.25 & 0 & 0 & 0.25 & 0.25 \\ 0 & 0 & 0.7 & 0 & 0.3 & 0 \\ 0 & 0.2 & 0 & 0.2 & 0.2 & 0.4 \end{bmatrix}.$$

1. Give the communication classes of the Markov chain and classify each state as recurrent or transient. No need to justify your answers.
2. Rewrite P in canonical form.
3. Suppose the system starts in state 1. What is the long term probability that it will be in state 1? Justify your answer without computation. Instead, use the canonical form and the formula derived at the end of Example 3.1 on page 78.
4. Suppose the system starts in state 6. What is the long term probability that it will be in state 6? Again, justify your answer without computation.

Problem 2. Try the following exercises from Chapter 3.

1. Exercise 3.13
2. Exercise 3.17 (closed form expression just means find a general formula for the entries of P^n ; use the recurrence and transience criteria in the box on page 98 to respond to the second part of the question)
3. Exercise 3.28 (make sure to justify the values of each entry in $\lim_{n \rightarrow \infty} P^n$ using the theory we've built up; it's possible to find each entry of the limiting matrix with very little computation)
4. Exercise 3.29 (we haven't yet talked about the period of each state formally so think about it and make a guess based on your intuition or reading so far)