

Math 339SP, Spring 2022 — Homework 6

Tim Chumley

Due March 25 at 5:00 pm

Instructions. This problem set covers material from Weeks 6 and 7 of class, with a focus on Chapter 3 of the textbook.

Problem 1. Give the limiting matrix $\lim_{n \rightarrow \infty} P^n$ in each case below for the example Markov chain in the lecture from March 8. See page 2 of the lecture notes for the transition state diagram.

1. $p = q = a = b = 0$
2. $p, q, a, b \neq 0$

Problem 2. Try the following exercises from Chapter 3.

1. Exercise 3.50, parts b and c. (This is an absorbing Markov chain problem in disguise. Remember that in card shuffling Markov chain models it's useful to think of the state space to be the set of permutations of the deck. That's why in a 3 card deck, there are $3! = 6$ states.)
2. Exercise 3.52 (Read the introduction to Section 3.8 for an overview of the Chutes and Ladders game. Note that if the player lands on square 2, they immediately move to square 7 and the next state of the Markov chain is 7, not 2. Similar rules apply to states 5 and 8. Also note that the finishing square, square 9, must be reached by an exact roll of the die, and the player stays in their current square if the die roll is higher than the exact number of steps needed.)