

Math 339SP, Spring 2024 — Homework 6

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Due March 15 at 5:00 pm

Instructions. This problem set covers material from Weeks 6 and 7 of class, with a focus on Chapter 3 of the textbook.

Problem 1. Consider the random walk on the graph shown below. Assume the random walk starts in state a . Use the theory of absorbing Markov chains to

1. Find the probability that state f is reached before state e .
2. Find the expected first hitting time of state f .

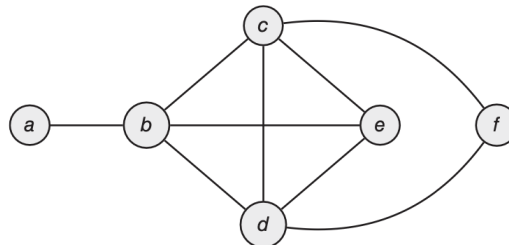


Figure 2.1 Graph on six vertices.

Problem 2. Consider the Markov chain with transition matrix given by

$$P = \begin{bmatrix} 1/12 & 1/12 & 4/12 & 2/12 & 3/12 & 1/12 \\ 5/15 & 3/15 & 1/15 & 1/15 & 3/15 & 2/15 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Find $\lim_{n \rightarrow \infty} P^n$ using the theory of absorbing Markov chains.

Problem 3. Let $p, q \in (0, 1)$ be arbitrary real numbers. Consider the Markov chain with transition matrix given by

$$P = \begin{bmatrix} 1/12 & 1/12 & 4/12 & 2/12 & 3/12 & 1/12 \\ 5/15 & 3/15 & 1/15 & 1/15 & 3/15 & 2/15 \\ 0 & 0 & 1-p & p & 0 & 0 \\ 0 & 0 & q & 1-q & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-a & a \\ 0 & 0 & 0 & 0 & b & 1-b \end{bmatrix}.$$

Find $\lim_{n \rightarrow \infty} P^n$ using your work from Problem 1 and the discussion in the [March 5 lecture notes](#).

Problem 4. Do the following book exercises.

1. Exercise 3.52 (Read the introduction to Section 3.8 for an overview of the Chutes and Ladders game. Note that if the player lands on square 2, they immediately move to square 7 and the next state of the Markov chain is 7, not 2. Similar rules apply to states 5 and 8. Also note that the finishing square, square 9, must be reached by an exact roll of the die, and the player stays in their current square if the die roll is higher than the exact number of steps needed.)
2. Exercise 3.56