Math 339SP, Spring 2024 — Homework 8

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Due April 5 at 5:00 pm

Instructions. This problem set covers material from Week 10 of class, with a focus on Chapters 6 and 7 of the textbook.

Problem 1. Try the following exercises from Chapter 6.

1. Exercise 6.7
2. Exercise 6.8d Note that in part d it will be helpful to use the continuous law of total probability: if X is a continuous random variable with density $f_X(x)$ taking on values in the interval $I \subseteq \mathbb{R}$ and A is an event, then $P(A) = \int_I P(A \mid X = x) f_X(x)\, dx$. *Hint: try letting X be the time between the third and fourth car arrival.*
3. Exercise 6.15
4. Exercise 6.16
5. Exercise 6.24.

Problem 2. A facility has three machines and one mechanic. Machines break down according to a Poisson process with a mean rate of one breakdown every 24 hours. The time it takes the mechanic to fix a machine is exponentially distributed with mean 6 hours, and they can only work on one machine at a time until it is fixed. Let X_t be the number of machines working at time t.

1. Give the state space of this Markov chain.
2. Draw the transition rate diagram.
3. Give the hold time parameters for each state.
4. Give the embedded chain transition matrix $\tilde{P}$.

Problem 3. Repeat the previous question but assume that there are 2 mechanics and only one mechanic can work on a failed machine at a time. That is, if only one machine is broken, only one mechanic will work. If at least two machines are broken, both mechanics will work independently on two machines.