Math 339SP — Absorbing chains

Problem 1. Every academic year, students at a 4-year college either drop out, repeat a year, or move on to the next year according to a random dynamic described by the Markov transition matrix

$$
P = \begin{array}{c}
\\ \text{Drop} \\ \text{Fr} \\ \text{So} \\ \text{Jr} \\ \text{Sr} \\ \text{Grad}
\end{array}
\begin{array}{c}
\begin{array}{cccccc}
\text{Drop} & \text{Fr} & \text{So} & \text{Jr} & \text{Sr} & \text{Grad}
\end{array} \\
\left[
\begin{array}{cccccc}
1 & 0 & 0 & 0 & 0 & 0 \\
0.06 & 0.03 & 0.91 & 0 & 0 & 0 \\
0.06 & 0 & 0.03 & 0.91 & 0 & 0 \\
0.04 & 0 & 0 & 0.03 & 0.93 & 0 \\
0.04 & 0 & 0 & 0 & 0.03 & 0.93 \\
0 & 0 & 0 & 0 & 0 & 1
\end{array}
\right]
\end{array}.
$$

a. Write P in canonical form and identify the submatrices corresponding to Q and R in our discussion on absorbing chains.

b. Consider a student who starts as a first-year. Download the file `absorption.R` from the class web page and use it to find the following quantities.

  1. Find the probability that they eventually graduate.

  2. Find the average number of years spent at the school until they graduate or drop out.

Problem 2. A gambler starts with $2 and plays a game where the chance of winning each round is 60%. The gambler either wins or loses $1 on each round. The game stops when the gambler either has $5 or is ruined (ie. has $0).

a. Write the transition matrix in canonical form.

b. Use R to compute the following quantities.

  1. Find the probability that the gambler is eventually ruined.

  2. Find the average number of rounds played until the game stops.