Math 339SP — Thinning and superposition

We've worked with questions that address a single Poisson process, but there are natural contexts in which this process gives rise to multiple independent (thinned) Poisson process, or when independent Poisson process should be combined into a single (superpositioned) Poisson process.

Problem 1. Starting at 6 a.m., cars, buses, and motorcycles arrive at a highway toll booth according to independent Poisson processes. Cars arrive about once every 5 minutes. Buses arrive about once every 10 minutes. Motorcycles arrive about once every 30 minutes. Let $(C_t)_{t\geq 0}, (B_t)_{t\geq 0}, (M_t)_{t\geq 0}$, and $(V_t)_{t\geq 0}$ denote the Poisson processes for cars, buses, motorcycles, and vehicles of any of these three types respectively, and let $\lambda_C, \lambda_B, \lambda_M, \lambda_V$ denote their mean rates.

a. State the values of $\lambda_C, \lambda_B, \lambda_M, \lambda_V$.

b. Find the probability that in the first 20 minutes exactly three vehicles arrive—two buses and one motorcycle.

c. Find the probability that in the first 20 minutes exactly three vehicles arrive.

d. Find the probability that the first vehicle to arrive is a car.

e. At the toll booth, the chance that a driver has exact change is $1/4$, independent of vehicle. Find the probability that no vehicle has exact change in the first 10 minutes.

f. Find the probability that it takes at least 20 minutes for 5 vehicles with exact change to arrive.

g. Find the probability that the seventh motorcycle arrives within 45 minutes of the third motorcycle.