Math 339SP — Hold times and embedded chains

For each of the following examples of continuous-time Markov chains (CTMC):

a. Draw a transition diagram showing the possible transitions that can occur between states. Don't label the edges yet.

b. Determine the hold-time parameter q_i for each state i.

c. Determine the transition matrix $\widetilde{P}$ for the embedded chain.

Problem 1. Consider a hair salon with two chairs—chair 1 and chair 2. A customer upon arrival goes initially to chair 1 where their hair is shampooed and rinsed. After this is done, the customer moves on to chair 2 where their hair is cut. The service times at the two chairs are assumed to be independent random variables that are exponentially distributed with respective rates μ_1 and μ_2. Suppose that potential customers arrive in accordance with a Poisson process with rate λ, and a potential customer will enter the system only if both chairs are empty. This is a CTMC with the following states.

State	Interpretation
0	salon is empty
1	a customer is in chair 1
2	a customer is in chair 2

Problem 2. At a small shop, there is one cash register where customers can check out, one at a time. Customers arrive in line to check out according a Poisson process with rate λ. The service time to check out each customer is independent from customer to customer and exponentially distributed with parameter μ. If the line is full (ie. there are 5 customers in line or checking out) an arriving customer will simply not get in line. This is a CTMC where the state space $\mathcal{S} = \{0, 1, 2, 3, 4, 5\}$ represents the number of customers in line or checking out.