Math 339SP — Infinitesimal generator

The following code shows an example of how to use R to compute $P(t) = e^{tQ}$ for a given time t.

```
library(expm)
Q = matrix(c(-3, 3,
             4, -4), nrow = 2, ncol = 2, byrow = TRUE)
expm(0.75*Q) # gives P(0.75)

##           [,1]      [,2]
## [1,] 0.5736775 0.4263225
## [2,] 0.5684300 0.4315700
```

Problem 1. Consider two independent machines that are maintained by a single person. Each machine functions for an exponentially distributed amount of time before breaking down. On average each machine functions for a half hour before breaking down. The repair time for either machine is exponentially distributed. The average repair time is 45 minutes. Assume that at time $t = 0$ (8:00 am) neither machine is broken. Find the following probabilities.

a. Both machines are broken at 10:30 am.

b. Neither machine is broken at 11:00 am.

c. One machine is broken at 2:15 pm.

d. The long term probabilities that 0, 1, or 2 machines are broken.

Problem 2. Consider a population where each member acts independently and takes an exponentially distributed amount of time, on average 6 months, to produce an offspring. Further, suppose that the lifespan of each member is exponentially distributed, with an average lifespan of 4 years. Finally suppose that when the population size is 5, offspring are no longer produced. Let X_t be the population size at time t in years and suppose that $X_0 = 1$. Find the distribution of the population size and its mean after

a. at 3 months

b. at 1 year

c. at 18 months