Math 339SP, Spring 2022 — Infinitesimal generator

Class on April 12

We have learned that the transition function $P(t)$ is given by $P(t) = e^{tQ}$. The following code shows how to use R to get $P(t)$ for a given generator Q and time t. Use similar code to do the exercises below.

```
library(expm)
Q = matrix(c(-3, 3,
             4, -4), nrow = 2, ncol = 2, byrow = TRUE)
expm(0.75*Q) # gives P(0.75)

##           [,1]      [,2]
## [1,] 0.5736775 0.4263225
## [2,] 0.5684300 0.4315700
```

Example 1 (April 7, Example 1)**.** Consider two independent machines that are maintained by a single person. Each machine functions for an exponentially distributed amount of time before breaking down. On average each machine functions for a half hour before breaking down. The repair time for either machine is exponentially distributed. The average repair time is 45 minutes. Assume that at time $t = 0$ (8:00 am) neither machine is broken. Find the following probabilities.

1. Both machines are broken at 10:30 am.
2. Neither machine is broken at 11:00 am.
3. One machine is broken at 2:15 pm.
4. The long term probabilities that 0, 1, or 2 machines are broken.

Example 2 (April 7, Example 2)**.** Repeat the previous example, assuming that there are 2 maintenance people. Recall that if only one machine is broken, one of them repairs it and the other is idle. If two machines are broken, then they can work simultaneously, but independently, on each machine.