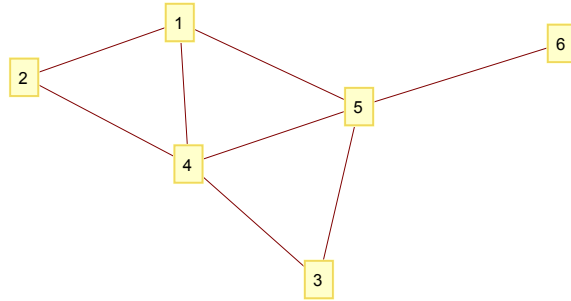


Math 339SP — Transition probabilities



Consider the following discrete-time Markov chain, called a **random walk on a graph**:

- A walker starts at a vertex of a given graph.
- At each time step, the walker moves to a neighbor vertex with each neighbor equally likely.
- Let X_n denote the walker's location after n steps.

Problem 1. Try the following exercises using the graph shown above.

- Find the transition matrix P .
- What do the following conditional probabilities mean in words? How many time units elapse? Between which states do you transition? What does the notion of *time-homogeneity* tell us about parts b and c?
 - $P(X_2 = 3 \mid X_0 = 1)$
 - $P(X_7 = 5 \mid X_4 = 4)$
 - $P(X_{50} = 5 \mid X_{40} = 2)$
- Try computing the probability in part 1 above using the Conditional Law of Total Probability. Wait until after doing Problem 2 before you think about the other probabilities.

Problem 2. Let's think more generally now. Suppose we're working with a Markov chain whose state space is $\mathcal{S} = \{1, 2, \dots, m\}$. Its transition matrix P is an $m \times m$ matrix and its ij -entry is $P_{ij} = P(X_1 = j \mid X_0 = i)$. This is all given to us. Try using the Conditional Law of Total Probability to write an expression for

$$P(X_2 = j \mid X_0 = i),$$

called the 2-step transition probability, in terms of the entries of the transition matrix P . Your expression should be a summation and its terms should be written using entries of the matrix P . Do you recognize this expression as something related to matrix algebra? If you figure these questions out, try thinking about $P(X_3 = j \mid X_0 = i)$, called the 3-step transition probability.