Math 339SP, Spring 2022 — Limiting distribution

Class on April 14

```
library(pracma)
Q = matrix(c(-3, 3,
             4, -4), nrow = 2, ncol = 2, byrow = TRUE)
aug.mat = cbind(t(Q), c(0,0)) # add column of 0's to Q^t
aug.mat = rbind(aug.mat, c(1,1,1)) # add row of 1's
aug.mat
```

```
##      [,1] [,2] [,3]
## [1,]   -3    4    0
## [2,]    3   -4    0
## [3,]    1    1    1
```

```
rref(aug.mat)
```

```
##      [,1] [,2]      [,3]
## [1,]    1    0 0.5714286
## [2,]    0    1 0.4285714
## [3,]    0    0 0.0000000
```

Example 1. During lunch hour, customers arrive at a fast-food restaurant at the rate of 120 customers per hour. The restaurant has one line, with three workers taking food orders at independent service stations. Each worker takes an exponentially distributed amount of time—on average 1 minutes—to service a customer. Assume that customers turn away from the store if all three service stations are busy. Let X_t denote the number of service stations busy at time t.

1. Find the generator Q.
2. Find the limiting distribution π.

Example 2. A facility has four machines, with two repair workers to maintain them. Individual machines fail on average every 10 hours. It takes an individual maintenance person on average 4 hours to fix a machine. Repair and failure times are independent and exponentially distributed.

1. Find the generator Q.
2. In the long-term, what is the expected number of operational machines?
3. If all four machines are initially working, find the probability that only two machines are working after 5 hours.