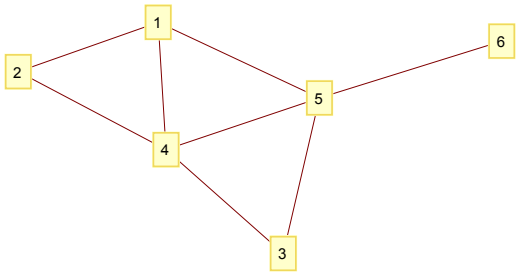
Math 339SP, Spring 2022 — Matrix powers

Class on February 1



Let's go back to the random walk on a graph from last time using the same graph, which is shown above.

Problem 1. Express the following probabilities in terms powers of the transition matrix P and its entries. Then use R to compute their values. The `matrix_powers.Rmd` file on the class web page gives a quick introduction to this.

1. $P(X_7 = 5 \mid X_4 = 4)$
2. $P(X_{50} = 5 \mid X_{40} = 2)$
3. $P(X_{12} = 3 \mid X_4 = 1)$

Problem 2. Use R to compute the matrices P^{25}, P^{50}, P^{75}.

1. Assume that the random walk starts at vertex 1. What do the matrix powers above tell you about the long term behavior of the random walk?
2. Suppose the random walk has been run for a large but finite number of steps. Which vertex is the most likely state for the random walk to be in at the end? Least likely?
3. What do you notice about the relationship between the long term probabilities and the degrees of each vertex? Note that the degree of a vertex is the number of neighbors it has.
4. You probably noticed the rows of P are all identical. What is the practical meaning of this?