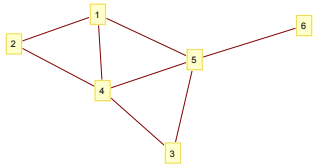
Math 339SP, Spring 2022 — Distributions

Class on February 3



Problem 1. Let's go back to the random walk on a graph from last time using the same graph, which is shown above. Suppose the random walker starts at a random vertex according to the probability vector $\alpha = (0.1, 0.2, 0.05, 0.35, 0.2, 0.1)$. By this we mean $P(X_0 = i) = \alpha_i$ for each $i = 1, \ldots, 6$. Express the following probabilities in terms α and P. Then use R to compute their values. The command `alpha = c(0.1,0.2,0.05,0.35,0.2,0.1)` will let you make a row vector.

1. $P(X_7 = 5)$
2. $P(X_{50} = 5, X_{40} = 2)$
3. $P(X_{12} = 3, X_4 = 1, X_2 = 3)$

Problem 2. Consider a Markov chain modeling a climate with three possible weather states: clear, rain, and snow. The day to day transition probabilities between states are given by

$$P = \begin{array}{c} \\ c \\ r \\ s \end{array} \begin{array}{c} \begin{array}{ccc} c & r & s \end{array} \\ \begin{bmatrix} 0.1 & 0.3 & 0.6 \\ 0 & 0.4 & 0.6 \\ 0.3 & 0.2 & 0.5 \end{bmatrix} \end{array}.$$

We also suppose that the initial distribution is $\alpha = (0.2, 0.3, 0.5)$. Use R to compute the following probabilities. Before doing so, convert the description into random variable notation, and then matrix notation.

1. The probability it's raining 5 days from now, given that it's snowing today.
2. The probability it's raining 5 days from now.
3. The probability it's raining 5 days from now and snowing 10 days from now, given that it's raining today.
4. The probability it's raining 5 days from now, snowing 10 days, and is clear 12 days from now, given that it's raining today.
5. The probability it's clear today, raining 5 days from now, snowing 10 days, and is clear 12 days from now.
6. The long term probabilities that it's clear, raining, or snowing.