Math 339SP — Limiting distributions

Problem 1. Consider the Markov chain with transition matrix

$$P = \begin{bmatrix} 1/4 & 3/4 \\ 2/3 & 1/3 \end{bmatrix}.$$

Use the formula for the limiting distribution of a 2-state Markov chain to find $\lim_{n\to\infty} P^n$ and verify your answer with R by computing P^{40}, P^{50}, P^{60}.

Problem 2. Consider the Markov chain with transition matrix

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/4 & 3/4 \\ 0 & 2/3 & 1/3 \end{bmatrix}.$$

a. Draw the transition state diagram for this Markov chain.

b. Does this Markov chain have a limiting *matrix*? That is, does $\lim_{n\to\infty} P^n$ exist? If so, compute it without using R, and then verify your answer with R. *Hint: notice this is a block-diagonal matrix.*

c. Does this Markov chain have a limiting *distribution*?

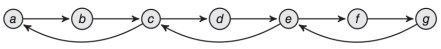
d. We will soon use the term *communication class* to describe a set of states. What do you think this term means? How many communication classes do you think this Markov chain has?

Problem 3. A *6-cycle* is a graph with 6 vertices that are arranged in a circle so that each vertex has two neighbors. Consider the random walk on a 6-cycle. Its transition matrix is given by

$$P = \begin{bmatrix} 0 & 1/2 & 0 & 0 & 0 & 1/2 \\ 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 \end{bmatrix}.$$

Does $\lim_{n\to\infty} P^n$ exist? Why or why not? Does this Markov chain have a limiting distribution?

Problem 4. Consider the Markov chain with transition state diagram below. Probabilities are missing in the diagram, but let's assume that for states c and e the probabilities for the two eminating arrows are each 1/2. Using R, what do you notice about powers of P? Does this Markov chain have a limiting *matrix*? Does this Markov chain have a limiting *distribution*? We will soon use the term *periodicity of a state.* What do you think it means? What do you think is the periodicity of each state in this chain.



Problem 5. Make a conjecture about two necessary conditions for a Markov chain to have a limiting distribution. Use the terms *communication class* and *periodicity.*