Math 339SP — Stationary distributions

Problem 1. Consider the Markov chain with transition matrix

$$P = \begin{bmatrix} 1/4 & 3/4 \\ 2/3 & 1/3 \end{bmatrix}.$$

a. It is not always the case that a Markov chain has a limiting distribution, but if it does, we've learned that the limiting distribution must be a stationary distribution. That is, it must satisfy the algebraic equation $\pi P = \pi$ where π is a row vector with two components that sum to 1. This yields a system with 2 unknowns (the two components of π) and 3 equations (why 3?). Write the system of 3 equations out and then express it as an augmented matrix.

b. Use the `rref.Rmd` file on the class webpage to find the *reduced row echelon form* of the augmented matrix you found. Convert what you get back into equations involving π_1 and π_2.

c. Compare your solution to what you get using the formula we learned last time for the limiting distribution of a 2-state Markov chain.

Problem 2. Consider the Markov chain with transition matrix

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 2/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Find all stationary distributions of P by solving $\pi P = \pi$. Use R to find the reduced row echelon form of an appropriate augmented matrix. Remember that sometimes a linear system of equations has infinitely many solutions and you need to introduce the notion of free variables to parametrize the solution set.

Problem 3. Repeat the previous problem using

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/3 & 2/3 & 0 & 0 \\ 0 & 0 & 4/5 & 1/5 \\ 0 & 0 & 1/4 & 3/4 \end{bmatrix}.$$

Problem 4. Use the same algebraic technique we have used in the rest of this worksheet to find all stationary distributions of the random walk on a 6-cycle. Compare with the general formula we learned for the stationary distribution of a random walk on a graph. Explain the advantage of the algebraic technique using the terms *existence* and *uniqueness* in your explanation.