

# Math 339SP, Spring 2022 — Stationary distributions

Class on February 10

**Problem 1.** Consider the Markov chain with transition matrix

$$P = \begin{bmatrix} 1/4 & 3/4 \\ 2/3 & 1/3 \end{bmatrix}.$$

1. We've learned that if a Markov chain has a limiting distribution, then the limiting distribution must be a stationary distribution. That is, it must satisfy the algebraic equation  $xP = x$  where  $x$  is a row vector with two components that sum to 1. This yields a system with 2 unknowns (the two components of  $x$ ) and 3 equations (why 3?). Write the system of 3 equations out and then express it as an augmented matrix.
2. Use the `rref.Rmd` file on the class webpage to find the *reduced row echelon form* of the augmented matrix you found. What do you notice in comparison to what you saw last time for this example?

**Problem 2.** Consider the Markov chain with transition matrix

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 2/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

1. Find all stationary distributions of  $P$  by solving  $xP = x$ . Use R to find the reduced row echelon form of an appropriate augmented matrix.
2. Later we'll learn the term *communication classes* of a Markov chain. What do you think is meant by this term and what do you think this example is hinting at about the relationship between communication classes and stationary distributions of Markov chains.

**Problem 3.** A *6-cycle* is a graph with 6 vertices that are arranged in a circle so that each vertex has two neighbors. Consider the random walk on a 6-cycle. Its transition matrix is given by

$$P = \begin{bmatrix} 0 & 1/2 & 0 & 0 & 0 & 1/2 \\ 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 \end{bmatrix}.$$

1. Find all stationary distributions of  $P$  by solving  $xP = x$ . Use R to find the reduced row echelon form of an appropriate augmented matrix.
2. Compare what you've found here with what you found out about this Markov chain last time. What are you curious about?