Math 342, Spring 2024 — Exam 1 revisions

Mount Holyoke College

Due March 29 at 5:00 pm

Instructions. As part of Exam 1, we will have a process for exam revisions. This will give you an opportunity to learn from mistakes and continue to understand the material more deeply. It will also allow you to earn back some credit on missed problems. I ask you to do the following:

a. For each problem that you lost points on

    1. redo the problem and give a correct solution.

    2. do all parts of the corresponding new problem below.

b. At the end of the revisions, write a short summary of went wrong in your original solutions and how you addressed the issues. This is open ended and about a paragraph long. It should be reflective and written in complete sentences.

This assignment is optional but will give you a chance to earn back up to 25% of points missed. For example, if your exam score was originally 40/50 and you complete the requirements above, your score will become 42.5/50. You can work with others and get help from me and TA's, as well as use the book and class notes.

Problem 1. For three events A, B, and C, the following is known: $P(\text{no events occur}) = P(\text{only } A) = P(\text{only } B) = P(\text{only } C)$, $P(AB) = P(AC) = 0.5P(\text{only } A)$, and $P(BC) = 0$.

a. Sketch a Venn diagram that matches the information provided about the three events.

b. Find

    1. $P(B)$

    2. $P(\text{exactly on event occurs})$

    3. $P(A \cup C \cup B^c)$

Problem 2. Suppose 100 customers enter the convenience store to buy 1 item and 35 of them buy a carbonated drink, 25 buy an uncarbonated drink, 20 buy a sweet snack, 15 buy a salty snack, and 5 buy a personal hygiene item. We decide to interview 5 of these customers at random and find out what they bought. Find the probability that

a. at least 3 of the customers you interview bought a drink of any kind

b. the customers you interview all bought different types of items

c. you interview three consecutive customers who bought sweet snacks

d. of the last two customers you interview, one bought a sweet snack and one bought a salty snack.

Problem 3. Suppose the number of particles emitted by source A is well-modeled by a Poisson (mean 4 particles per hour) distribution. Source B also emits particles at an average rate of 7 particles per hour according to the Poisson distribution. You are going to be given data about the number of particles emitted in a given hour, but you're not sure of the source. All you know is that there's a 30% chance it will come from source A and a 70% chance it came from B.

a. If the data comes from B, what is the probability of seeing at least 8 particles emitted?

b. Suppose you saw exactly 8 particles emitted in the data. Find the probability it came from B.

Problem 4. In a certain court system, the decision of whether the defendant is declared guilty of a crime is determined as follows. A jury of 12 people listens to the case and each is then independently asked to vote whether the defendant is guilty or innocent. The defendant will then be convicted of the crime if at least 9 members of the jury vote guilty. The issue is that jury members cannot perfectly judge whether someone is guilty or innocent. The probability that a juror will vote "innocent" for an innocent person is 0.2, and the probability that a juror votes "innocent" for a guilty person is 0.1. Find the probability that

a. An innocent person is convicted.

b. A guilty person is not convicted.

Problem 5. A four-sided die is labeled with numbers $\{1, 2, 3, 4\}$ and rolled 25 times. Let X_1 and X_2 be the result of the first and second rolls respectively, and let $X = \max\{X_1, X_2\}$.

a. Find the probability mass function of X.

b. Find $E[X]$.

c. Find $E[3X^2 + 7X - 5]$.

d. Find the probability that 2 randomly chosen rolls sum to a value no more than 3.

e. Find the probability that 3 randomly chosen rolls sum to a value no more than 4.