Math 342, Spring 2024 — Exam 2 revisions

Mount Holyoke College

Due May 3 at 5:00 pm

Instructions. As part of Exam 2, we will have a process for exam revisions. This will give you an opportunity to learn from mistakes and continue to understand the material more deeply. It will also allow you to earn back some credit on missed problems. I ask you to do the following:

a. For each problem that you lost points on

   1. redo the problem and give a correct solution.
   2. do all parts of the corresponding new problem below.

b. At the end of the revisions, write a short summary of went wrong in your original solutions and how you addressed the issues. This is open ended and about a paragraph long. It should be reflective and written in complete sentences.

This assignment is optional but will give you a chance to earn back up to 25% of points missed. For example, if your exam score was originally 40/53 and you complete the requirements above, your score will become 43.25/53. You can work with others and get help from me and TA's, as well as use the book and class notes.

Problem 1. Suppose that X and Y are i.i.d. discrete random variables with probability mass function given by

$$f(k) = \begin{cases} 1/3 & k = -3 \\ 1/2 & k = 5 \\ 1/6 & k = 6 \end{cases}$$

a. Find the variance of $X - 7Y$.

b. Find the moment generating function of X.

c. Find the moment generating function of $X - 7Y$.

Problem 2. For each of the following discrete distributions, describe a real-world scenario and random variable which has that distribution and give the corresponding parameters. Think of this as writing a new exam problem like the one that appeared on Exam 2.

a. Binomial

b. Hypergeometric

c. Poisson

d. Geometric

e. Negative Binomial

Problem 3. Suppose that X is a continuous random variable with cumulative distribution function denoted by $F(x)$ and probability density function given by

$$f(x) = \begin{cases} \frac{2}{\pi}\sqrt{1-x^2} & -1 < x \leq 0 \\ \frac{1}{2} & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

a. Find $P(X < 0.5)$.

b. Find $E[X]$.

c. Find $F(0)$.

d. Find a formula for $F(x)$ when x is an arbitrary value such that $0 < x < 1$.

Problem 4. Suppose that X is a continuous random variable with probability density function denoted by $f(x)$ and cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x < 1 \\ ax + b & 1 \leq x \leq 3 \\ \frac{1}{12}x + \frac{1}{4} & 3 < x < 9 \\ c & x \geq 9 \end{cases}$$

for some constants $a, b, c \in \mathbb{R}$.

a. Find a, b, c.

b. Find $P(2 < X < 5)$.

c. Find $f(x)$. Make sure to give your answer as a piecewise function.

d. Find $E[X]$.

Problem 5. Suppose that someone receives emails according to a Poisson process and on average the time between emails is 4 hours. Let X denote the waiting time, in hours, between emails.

a. Give the distribution of X, including any relevant parameters.

b. Find the probability that it takes more than half a day until the next email arrives.

c. Given that an email has not arrived after 4 hours of waiting, find the conditional probability that an email will not be received after waiting an additional 2 hours.