Math 342, Spring 2024 — Homework 6

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Due March 15 at 5:00 pm

Instructions. This problem set contains problems from Weeks 6 and 7 of class. The problem numbers refer to our textbook, *Probability with Applications and R*, by Amy Wagaman and Robert Dobrow, 2nd edition.

Problem 1. Do the following textbook problems and submit on Gradescope: 5.13, 5.14, 5.17, 5.30 (see Section 5.4 for the formula for the variance of a hypergeometric random variable), 5.32, 5.34, 6.4.

Problem 2. Let X be a random variable with probability density function f given by

$$f(x) = \begin{cases} 3x^2 & 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

1. Plot $f(x)$ and find a formula for $F(x) = P(X \leq x)$ for the following cases of x. Your final answer should be a piecewise function of the form

$$F(x) = \begin{cases} \dots & x \leq 0 \\ \dots & 0 < x < 1 \\ \dots & x \geq 1. \end{cases}$$

   (a) $x \leq 0$
   (b) $0 < x < 1$
   (c) $x \geq 1$

2. Using the formula you found for $F(x)$, doing no more integration, find

   (a) $P(X \leq 1/3)$
   (b) $P(X > 1/2)$
   (c) $P(1/4 \leq X \leq 3/4)$

Problem 3. Let X be a random variable whose probability density function is proportional to x^{-4} for $x > 1$. That is,

$$f(x) = \begin{cases} cx^{-4} & x > 1 \\ 0 & x \leq 1. \end{cases}$$

1. Plot $f(x)$ and find c.

2. Find a formula for $F(x) = P(X \leq x)$ for the following cases of x. Your final answer should be a piecewise function of the form

$$F(x) = \begin{cases} \dots & x \leq 1 \\ \dots & x > 1 \end{cases}$$

   (a) $x \leq 1$
   (b) $x > 1$.

3. Using the formula you found for $F(x)$, doing no more integration, find

   (a) $P(3 < X < 6)$,

   (b) $P(2 < X < 3)$,

   (c) $P(X \geq 4)$

Problem 4. A random variable X has density

$$f(x) = \begin{cases} ce^{x} & -2 < x \leq 0 \\ ce^{-x} & 0 < x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

1. Plot $f(x)$ and find c.

2. Find a formula for $F(x) = P(X \leq x)$ for the following cases of x. Your final answer should be a piecewise function of the form

$$F(x) = \begin{cases} \dots & x \leq -2 \\ \dots & -2 < x \leq 0 \\ \dots & 0 < x \leq 2 \\ \dots & x > 2. \end{cases}$$

   (a) $x \leq -2$

   (b) $-2 < x \leq 0$

   (c) $0 < x \leq 2$

   (d) $x > 2$.

3. Find $P(-1 < X < 1)$ using the formula you found for $F(x)$.

Problem 5. If you liked the problems above or want more practice, our textbook has more great problems. The odd-numbered ones have solutions in the back. Here are some that I recommend (as optional, not to be turned in): 6.1, 6.3, 6.5, 6.7. Feel free to try others, including all the problems in the main sections, which include full explanations.