

# Math 342, Fall 2024 — Homework 7

Tim Chumley

Due November 1 at 5:00 pm

**Instructions.** This problem set contains problems from Week 8 of class. The problem numbers refer to our textbook, *Probability with Applications and R*, by Amy Wagaman and Robert Dobrow, 2nd edition.

**Problem 1.** Do the following textbook problems and submit on Gradescope: 6.4 (the notation  $\exp(x)$  means  $e^x$ ), 6.6, 6.7, 6.10 (to show  $f$  is a probability density function, explain why  $f(x) \geq 0$  for all  $x \in \mathbb{R}$  and show that  $\int_{-\infty}^{\infty} f(x) dx = 1$ ).

**Problem 2.** Let  $X$  be a random variable with probability density function  $f$  given by

$$f(x) = \begin{cases} 3x^2 & 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

- a. Plot  $f(x)$  and find a formula for  $F(x) = P(X \leq x)$  for the following cases of  $x$ . Your final answer should be a piecewise function of the form

$$F(x) = \begin{cases} \dots & x \leq 0 \\ \dots & 0 < x < 1 \\ \dots & x \geq 1. \end{cases}$$

1.  $x \leq 0$
  2.  $0 < x < 1$
  3.  $x \geq 1$
- b. Using the formula you found for  $F(x)$ , doing no more integration, find
1.  $P(X \leq 1/3)$
  2.  $P(X > 1/2)$
  3.  $P(1/4 \leq X \leq 3/4)$

**Problem 3.** Let  $X$  be a random variable whose probability density function is proportional to  $x^{-4}$  for  $x > 1$ . That is,

$$f(x) = \begin{cases} cx^{-4} & x > 1 \\ 0 & x \leq 1. \end{cases}$$

- a. Plot  $f(x)$  and find  $c$ .
- b. Find a formula for  $F(x) = P(X \leq x)$  for the following cases of  $x$ . Your final answer should be a piecewise function of the form

$$F(x) = \begin{cases} \dots & x \leq 1 \\ \dots & x > 1 \end{cases}$$

1.  $x \leq 1$
2.  $x > 1$ .

c. Using the formula you found for  $F(x)$ , doing no more integration, find

1.  $P(3 < X < 6)$ ,
2.  $P(2 < X < 3)$ ,
3.  $P(X \geq 4)$

**Problem 4.** If you liked the problems above or want more practice, our textbook has more great problems. The odd-numbered ones have solutions in the back. Here are some that I recommend (as optional, not to be turned in): 6.1, 6.3, 6.5, 6.9. Feel free to try others, including all the problems in the main sections, which include full explanations.