

Math 342, Spring 2024 — Homework 9

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Due April 19 at 5:00 pm

Instructions. This problem set contains problems from Week 12 of class. The problem numbers refer to our textbook, *Probability with Applications and R*, by Amy Wagaman and Robert Dobrow, 2nd edition.

Problem 1. Do the following textbook problems and submit on Gradescope: 8.2, 8.4, 8.6a, 8.29, 8.30, 8.31. *Note:* when doing Exercise 8.4, it will be helpful to recall that

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}.$$

Fun fact: the density you find in this exercise appeared in my PhD thesis. It's called the Cauchy distribution.

Problem 2. Consider the probability density function

$$f(x) = \begin{cases} \frac{1}{(1+x)^2} & x > 0 \\ 0 & x \leq 0, \end{cases}$$

which has cumulative distribution function

$$F(x) = \int_0^x \frac{1}{(1+t)^2} dt = \frac{x}{x+1}$$

when $x > 0$.

- Find the inverse function $F^{-1}(x)$ by solving the equation $x = \frac{y}{y+1}$ for y .
- Let $U \in \text{Unif}(0,1)$. Define a new random variable Y whose density is f . Express Y as a transformation of U .

Problem 3. Consider the probability density function

$$f(x) = \begin{cases} \frac{4}{3x^2} & 1 < x < 4 \\ 0 & \text{otherwise.} \end{cases}$$

- Find the corresponding cumulative distribution function $F(x)$ for $1 < x < 4$ by computing

$$\int_1^x \frac{4}{3t^2} dt.$$

- Find the inverse function F^{-1} of F .
- Compute $\int_1^4 xf(x) dx$.
- Let $Y = F^{-1}(U)$ where $U \sim \text{Unif}(0,1)$. Write code in the [hw9.Rmd](#) RMarkdown file and knit to PDF after doing the following. Submit this PDF as part of your homework submission.

1. Use R to simulate 1 million samples of Y . Do not change the first two lines of code chunk in the linked RMarkdown file.
 2. Make a histogram of your simulated samples of Y and overlay the density f on the histogram.
 3. Use the `mean` command to find the average of your 1 million samples.
- e. Give a brief informal explanation for why your answer to part c. is roughly equal to the last part of your work in part d.

Problem 4. If you liked the problems above or want more practice, our textbook has more great problems. The odd-numbered ones have solutions in the back. Here are some that I recommend (as optional, not to be turned in): 8.3, 8.5, 8.8, 8.17, 8.33ab, 8.34. Feel free to try others, including all the problems in the main sections, which include full explanations.