\$ 1.7 Counting I

Example Consider the 7-element at {A, B, C, D, E, F, G}.

- (1) How many 4-letter words can we make, assuming no repeated letters? (ie we're making an ordered sequence, sampling without replacement) $7 \times 6 \times 5 \times 4 = \frac{7!}{3!}$
- (2) How many 4-element subsets over there of the 7-element set? (With subsets, the order of the elements does not matter)

of 4-letter =
$$\frac{\text{# of 4-letter}}{\text{ordered sequences}}$$
 = $\frac{7!}{3!}$ = $\frac{7!}{3!4!}$ of permutations of the 4 letters

General Principle Given an n-element set,

- ① The number of k-element ordered sequences with no repitition is $\frac{h!}{(n-k)!}$
- (2) The number of k-element unordered subsets $is \frac{n!}{(n-k)!k!} = \binom{n}{k}$ "n choose k" Called a "binomial coefficient"

Example 25 people participate in a clinical trial.

15 will receive treatment

10 will receive placebo.

Consider a group of 6 people in the trial chosen at random. What is the probability of received treatment (and 2 received the placebo)?

of groups of 6

where 4 receive treatment =
$$\frac{\binom{15}{4}\binom{10}{2}}{\binom{25}{6}}$$

Example 5 cards are chosen from a 52 card deck. What is the probability of getting 1) 3 K's, 2 Q's?

2) 3 K's?

$$() \qquad \frac{\binom{4}{3}\binom{4}{2}}{\binom{52}{5}} \qquad 2) \qquad \frac{\binom{4}{3}\binom{49}{2}}{\binom{52}{5}}$$

Example A person has 8 friends, 5 will be invited to a party.

Suppose Alice and Bub are two of the friends.

What is the prob. A and B are both invited? $\frac{\binom{2}{2}\binom{6}{3}}{\binom{8}{5}}$

What is the prob. A is invited but B is not? $\frac{\binom{1}{1}\binom{1}{0}\binom{6}{24}}{\binom{8}{5}}$

Problem 1. An urn contains 20 balls; 10 are green, 6 are white, and 4 are red. Suppose we draw 7 balls at random, without replacement. Find the probability of each of the following events.

- a. We choose exactly 4 green balls.
- b. We choose 4 green balls and 3 white balls.
- c. We choose 2 green, 3 white, and 2 red balls.

 $\textbf{Problem 2.} \ \, \textbf{Consider tossing a coin 6 times.} \ \, \textbf{Use binomial coefficients to express the number of outcomes in each of the following events.}$

- a. Exactly 3 of the flips is heads.
- b. Exactly 2 of the flips is heads.
- c. Exactly 1 of the flips is heads.
- d. There are no heads.

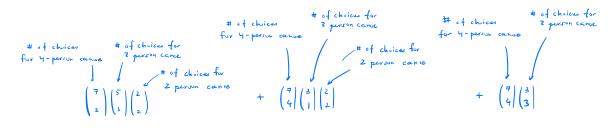
Problem 3. We choose 5 cards from the standard deck of cards. Find the probability of each of the following events.

- a. Our draw contains exactly one ace. Note that a deck of 52 cards contains 4 aces.
- b. Our draw contains at least one ace.
- c. Our draw contains exactly 2 aces.

 $\textbf{Problem 4.} \ A \ committee \ of \ 7 \ is \ to \ be \ chosen \ at \ random \ from \ a \ group \ of \ 5 \ Republicans, \ 6 \ Democrats, \ and \ 4 \ Independents. Find the probability that the committee contains:$

- a. 2 Republicans, 2 Democrats, and 3 Independents.
- b. at least 2 independents.

Problem 5. Alice, Bob, and 7 friends will go on a canoe trip. They'll have 3 canoes that hold 2, 3 and 4 people respectively, and will divide themselves randomly with every arrangement equally likely. What is the probability Alice and Bob are on the same canoe? (Can you break this down by thinking about the probability they're on the same 4 person canoe, the same 3 person canoe, and the same 2 person canoe separately?)



 $\begin{pmatrix} 9 \\ 4 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix}$