Math 342 —Expectation

Problem 1. Suppose a person always runs a mile in either 4 minutes, 5 minutes, 6 minutes, 10 minutes, or 15 minutes, uniformly at random. Let T denote the time of their run, in minutes, on a given day.

a. Find $E[T]$.

b. Let τ be the time of their run in hours. Find $E[\tau]$.

c. Let S be their speed in miles per hour. Find $E[S]$.

Problem 2. Consider the following gambling game, which costs \$7 to play. You toss a coin 5 times. If the coin comes up heads fewer than 3 times, you lose your money. If the coin comes up heads 3 times you get your money back. The coin comes up heads 4 times, you win \$10. If the coin comes up heads 5 times, you win \$50. Let W represent your net winnings.

a. Find the range of W.

b. Find the probability mass function of W.

c. Find $E[W]$.

Problem 3. Suppose

- $X \sim \text{Unif}\{1, 7\}$. This means $P(X = k) = 1/2$ for $k = 1, 7$.
- $Y \sim \text{Unif}\{1, 2, 3, 4, 5, 6, 7\}$. This means $P(Y = k) = 1/7$ for $k = 1, \ldots, 7$.
- $Z = 4$. This means Z is a constant; ie. $P(Z = 4) = 1$.

Find the following quantities

a. $E[X]$

b. $E[Y]$

c. $E[Z]$

d. $E[X^2]$

e. $E[Y^2]$

f. $E[Z^2]$