Math 342 —Expectation

Problem 1. Suppose a person always runs a mile in either 4 minutes, 5 minutes, 6 minutes, 10 minutes, or 15 minutes, uniformly at random. Let T denote the time of their run, in minutes, on a given day.

a. Find $E[T]$.

b. Let S be their speed in miles per minute.

   1. Express S in terms of T.
   2. Find $E[S]$ using the Law of the Unconscious Statistician.

Problem 2. Let $p \in (0,1)$ and suppose $X \sim \mathrm{Ber}(p)$. Recall that this means X is a random variable whose range is $\{0,1\}$ and $P(X=1)=p, P(X=0)=1-p$. Find the following quantities:

a. $E[X]$.

b. $E[X^2]$.

c. $E[5X+3]$

d. $E[4X^2-2]$

e. $E[\sin(\pi X+\pi/2)]$

Problem 3. Consider the following gambling game, which costs \$7 up front to play. You toss a coin 5 times. If the coin comes up heads fewer than 3 times, you get nothing back. If the coin comes up heads 3 times you get your money back. The coin comes up heads 4 times, you get \$10 back. If the coin comes up heads 5 times, you get \$50 back. Let W represent your net winnings.

a. Find the range of W.

b. Find the probability mass function of W.

c. Find $E[W]$.

Problem 4. Suppose

- $X \sim \mathrm{Unif}\{1,7\}$. This means $P(X=k)=1/2$ for $k=1,7$.
- $Y \sim \mathrm{Unif}\{1,2,3,4,5,6,7\}$. This means $P(Y=k)=1/7$ for $k=1,\ldots,7$.
- $Z=4$. This means Z is a constant; ie. $P(Z=4)=1$.

Find the following quantities

a. $E[X]$

b. $E[Y]$

c. $E[Z]$

d. $E[X^2]$

e. $E[Y^2]$

f. $E[Z^2]$