## Math 342 -Joint probability mass functions

Problem 1. Suppose you have the following data on pet ownership in a town with 1000 households.

|  | Has 0 cats | Has 1 cat | Has 2 cats | total |
| :---: | :---: | :---: | :---: | :---: |
| Has 0 dogs | 400 | 150 | 150 | 700 |
| Has 1 dog | 50 | 5 | 40 | 95 |
| Has 2 dogs | 150 | 15 | 40 | 205 |
| total | 600 | 170 | 230 | 1000 |

Suppose a household is chosen at random, and we let $X$ denote the number of dogs in the household and let $Y$ denote the number of cats in the household.
a. Make a table for the joint probability mass function of $X$ and $Y$.
b. Find the marginal probability mass functions of $X$ and $Y$.
c. Find $E[X]$ and $E[Y]$.

Problem 2. Suppose we draw two numbers, one at a time without replacement, from the set $\{1,2,3,4\}$. Let $X$ denote the first number drawn, and let $Y$ denote the second number drawn.
a. For each value of $x$ and $y$, compute $P(X=x, Y=y)$ by computing $P(Y=y \mid X=x) P(X=$ $x)$. Write your answers in a table.
b. Find the marginal probability mass functions of $X$ and $Y$.
c. Find $E[X]$ and $E[Y]$.
d. Compute $E[X Y]$. How does it compare to $E[X] E[Y]$ ? Is this surprising?
e. How does $E[X+Y]$ compare to $E[X]+E[Y]$ ?

