Math 342 —Variance and linearity of expectation

Problem 1. Suppose we play a game, which costs $10 to play, based on the experiment of rolling a die. If our roll is 3 or less, we lose our entry fee. If our rolls is 4, we get our entry fee back. If we roll a 5 or 6, we get our entry fee back along with $12. Find the expected value, variance, and standard deviation of our net winnings.

Problem 2. Suppose $E[X] = a, E[Y] = b, E[X^2] = c$, and $E[Y^2] = d$. Express the following in terms of a, b, c, d.

    a. $E[3X - 4Y]$

    b. $E[X^2 - 7Y^2 + 3X]$

    c. $V(-4X + 1)$

    d. $V(2X - 3Y + 4)$, assuming X and Y are independent

Problem 3. The following small algebra exercises lead up to proving the formula $V(X + Y) = V(X) + V(Y)$ when X and Y are independent.

    a. Express $E\left[(X + Y)^2\right]$ as a sum of three expectations.

    b. Let $\mu_X = E[X]$ and $\mu_Y = E[Y]$. Express $(\mu_X + \mu_Y)^2$ as a sum of three terms involving μ_X and μ_Y.

    c. Show that when X and Y are independent, $V(X + Y) = V(X) + V(Y)$.