

Math 342 —Moment generating functions

Problem 1. Let X be a random variable whose probability mass function is given by

$$P(X = k) = \begin{cases} 1/6 & k = -2 \\ 1/3 & k = 0 \\ 1/2 & k = 3. \end{cases}$$

- Find the moment generating function $m(t)$ of X .
- Compute $E[X]$ and $E[X^2]$ using your answer to part a.
- Suppose Y is independent of X with the same distribution. Find the moment generating function of $X + Y$ using your answer to part a.

Problem 2. Let $X \sim \text{Ber}(p)$. Find the moment generating function of X and the first three moments of X .

Problem 3. Let $Y \sim \text{Bin}(n, p)$. Use the moment generating function you found in Problem 2 to find the moment generating function of Y . Remember that $Y = X_1 + X_2 + \cdots + X_n$ where $X_1, X_2, \dots, X_n \sim \text{Ber}(p)$ are i.i.d.

Problem 4. Let $Y \sim \text{Bin}(n, p)$ and $Z \sim \text{Bin}(m, p)$ be independent random variables. Use the moment generating function you found in Problem 3 to find the moment generating function of $Y + Z$ and then determine the distribution of $Y + Z$, including any relevant parameters.